Vortex wake models with application to yawed rotor

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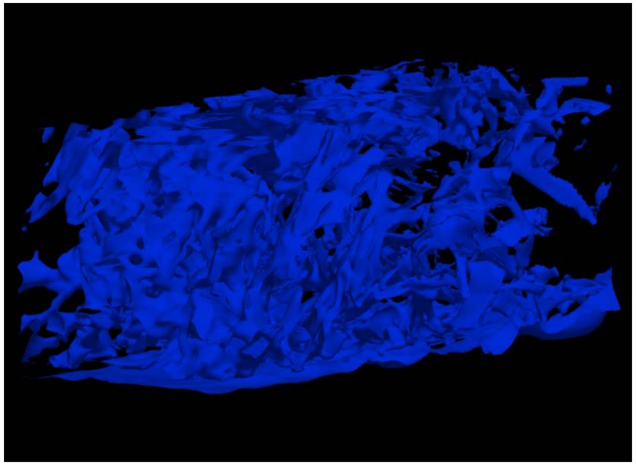
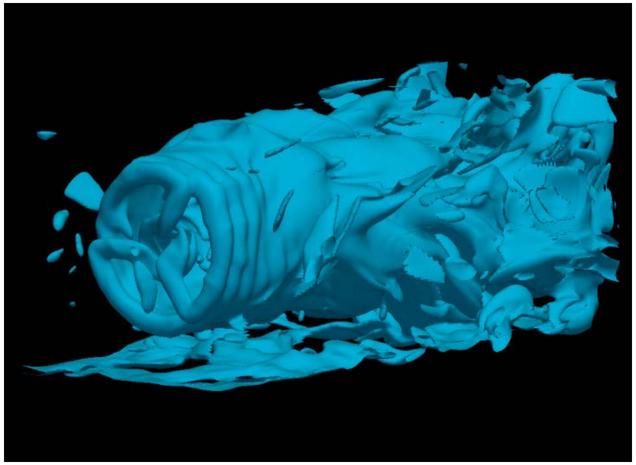
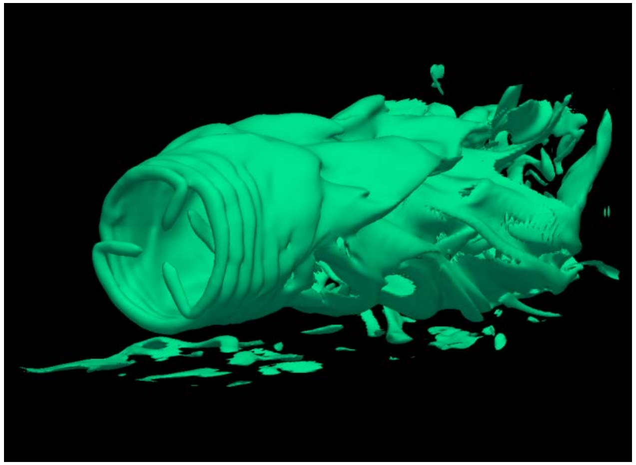
DTU Wind Energy
Department of Wind Energy

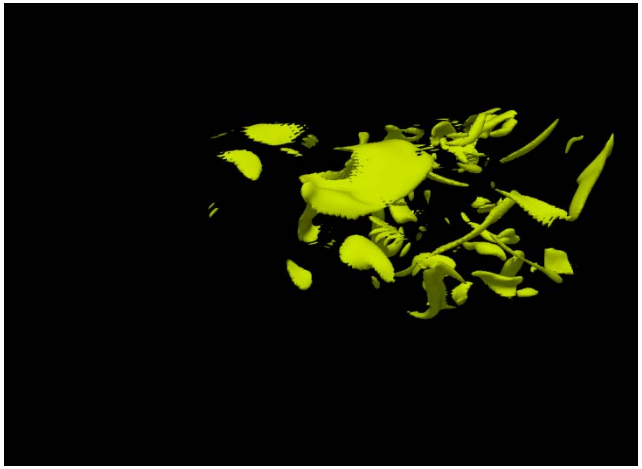
Introduction – Vorticity

Iso-vorticity contours

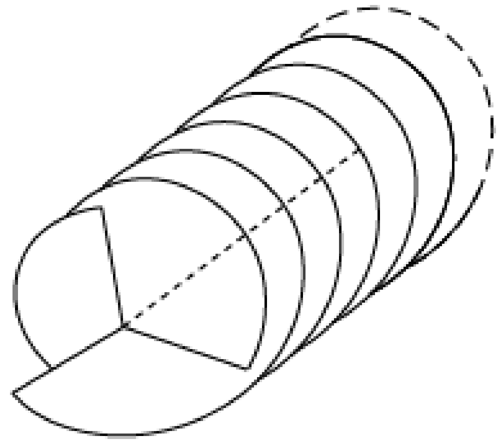


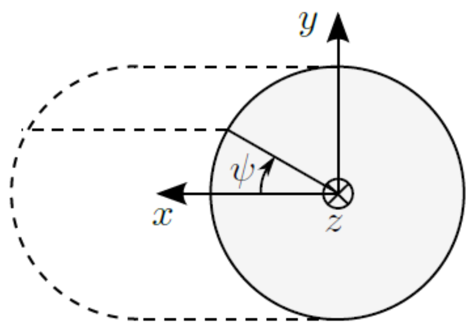




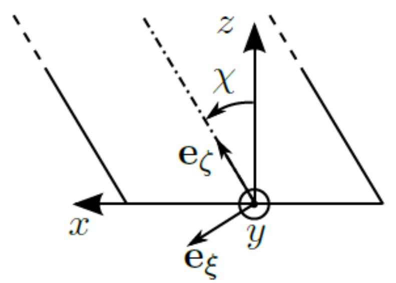


1. Presentation of the model





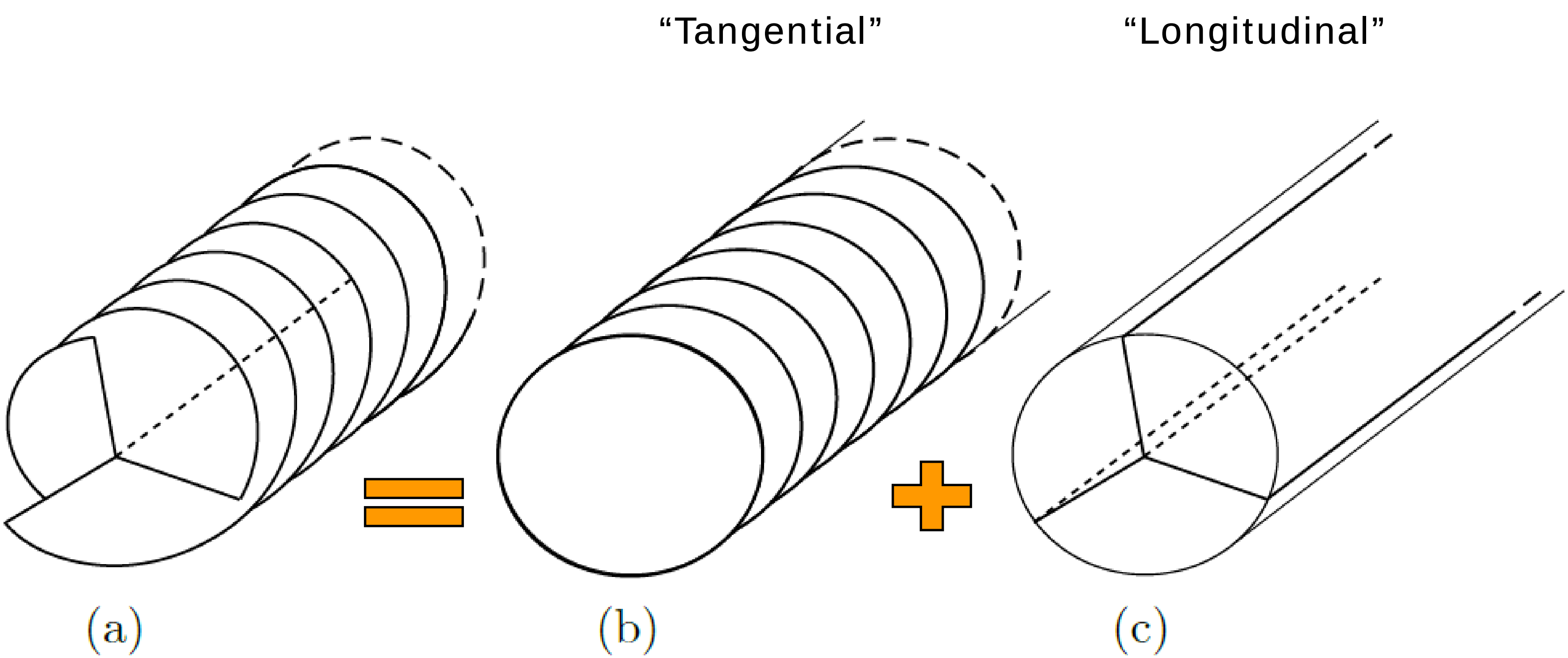
Rotor plane/Front view



Wake/Top view

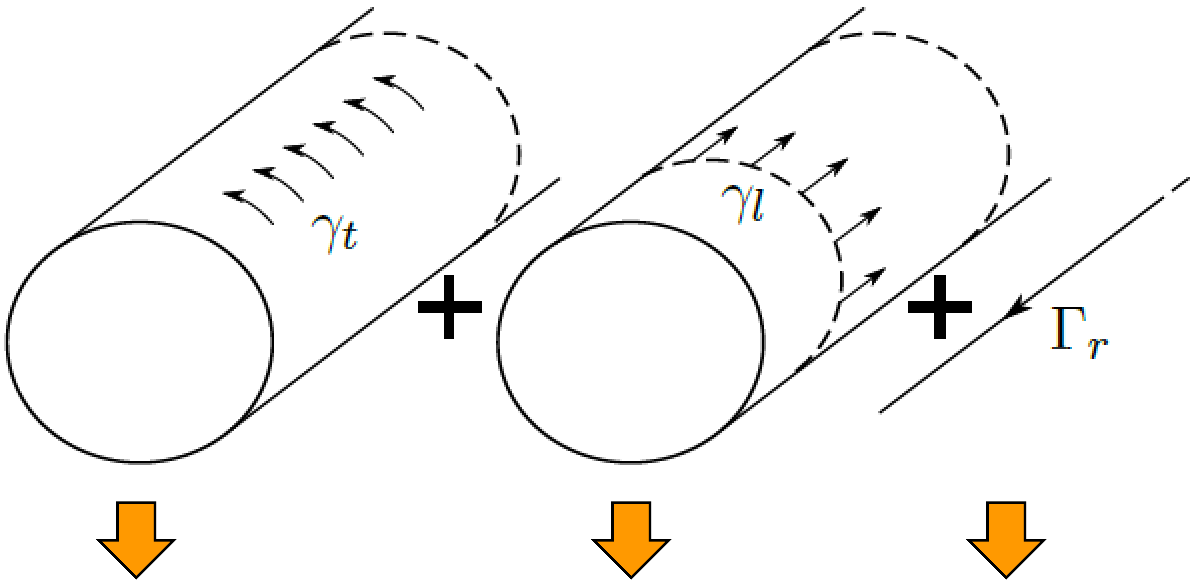
1. Presentation of the model

Decomposition of (skewed) helical wake



1. Presentation of the model

Viewed with infinite number of blades



Coleman(1945)
Castles (1956)
Current study

ECN models(1995)
Current Study

ECN models (1995)
WEH
Current Study

2. Methodology for further investigations

Biot-savart law – Integration over z

$$I = C\int_0^{2\pi}\int_0^{\infty}\frac{a' + b'z}{(a+bz+cz^2)^{3/2}}dz d\theta = C\int_0^{2\pi} I_z \, d\theta$$

(Pierce, 1899)

$$I_z = \frac{1}{\sqrt{c}} + \frac{2(2ab' - a'b)}{\sqrt{a}\,(4ac-b^2)} + \frac{4c(a'-a) + b(b-2b')}{\sqrt{c}\,(4ac-b^2)}$$

(Suitable for analytical expressions - Work of Coleman)

$$I_z = \frac{2\left(a'\sqrt{c} + b'\sqrt{a}\right)}{\sqrt{ac}\,(2\sqrt{ac}+b)}$$

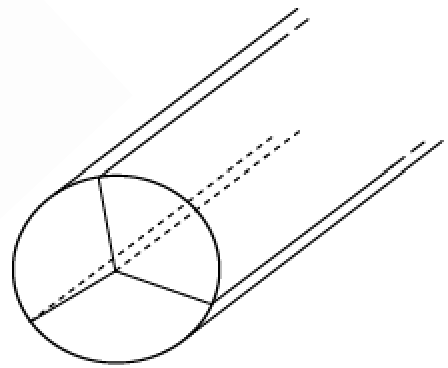
("Suitable" for numerical integration - Work of Castles)

2. Methodology for further investigations

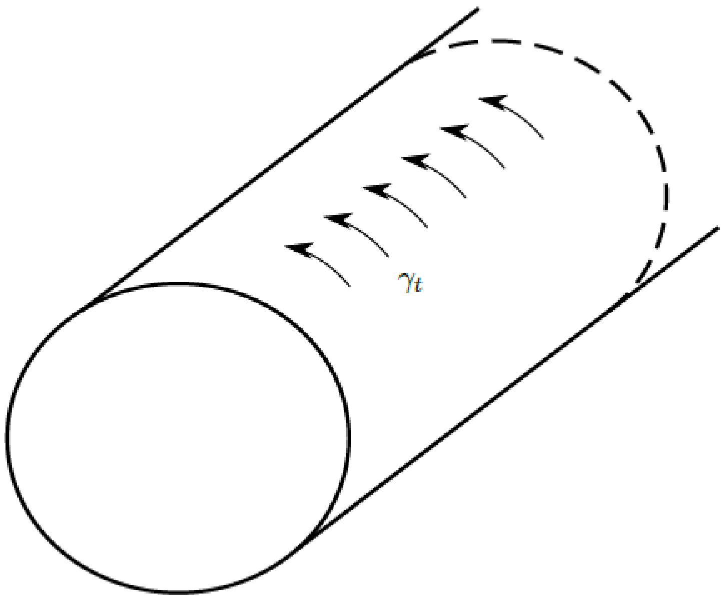
Longitudinal vorticity – semi-infinite lines

Biot-Savart law for semi-infinite line:

$$\mathbf{u}(\mathbf{x}) = \frac{\Gamma}{4\pi} \frac{\mathbf{e} \times r_1}{r_1 \left(r_1 - \mathbf{e} \cdot \mathbf{r}_1\right)}$$



3. Tangential vorticity

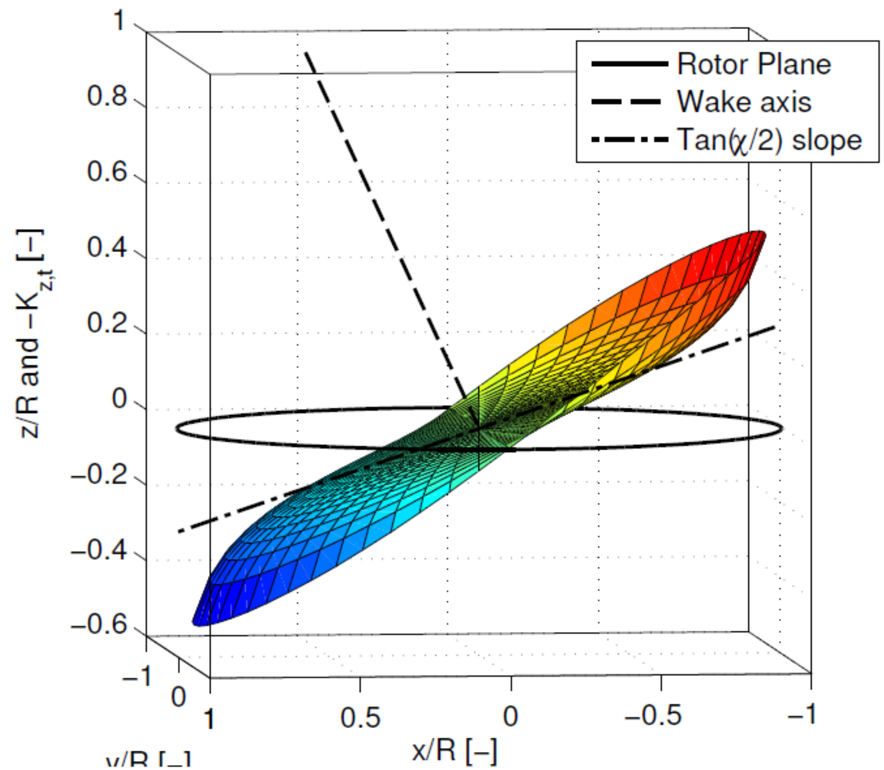


3. Tangential vorticity

Axial component

$$u_{z,t}(r,\psi,\chi) = u_{z,0}\left(1 + K_{z,t}(r,\chi)\sin\psi\right)$$

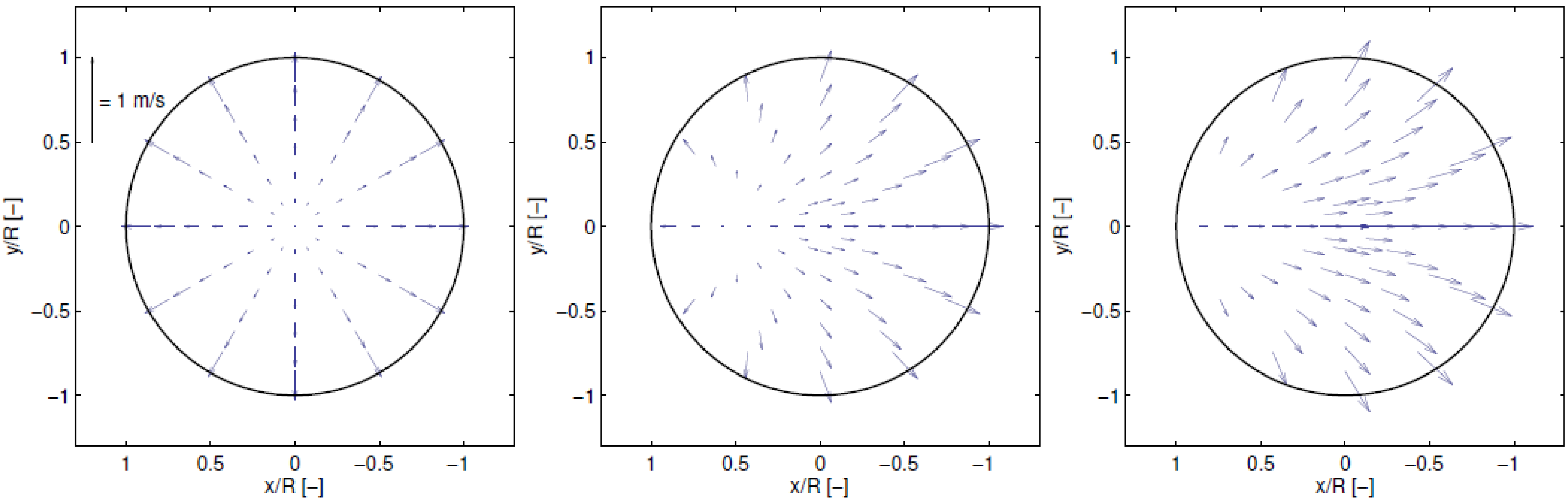
$$K_{z,t,\mathrm{approx}} \approx \left.\frac{\partial K_{z,t}}{\partial r}\right|_{r=0} = \frac{r}{R}\tan\frac{\chi}{2}$$



Incoming wind

3. Tangential vorticity

In-plane component for various skew angles



(a) $\chi = 0°$ (b) $\chi = 30°$ (c) $\chi = 60°$

$$u_{\psi,t,\text{approx}} = -u_{z,0} \tan\frac{\chi}{2} \sin\psi$$

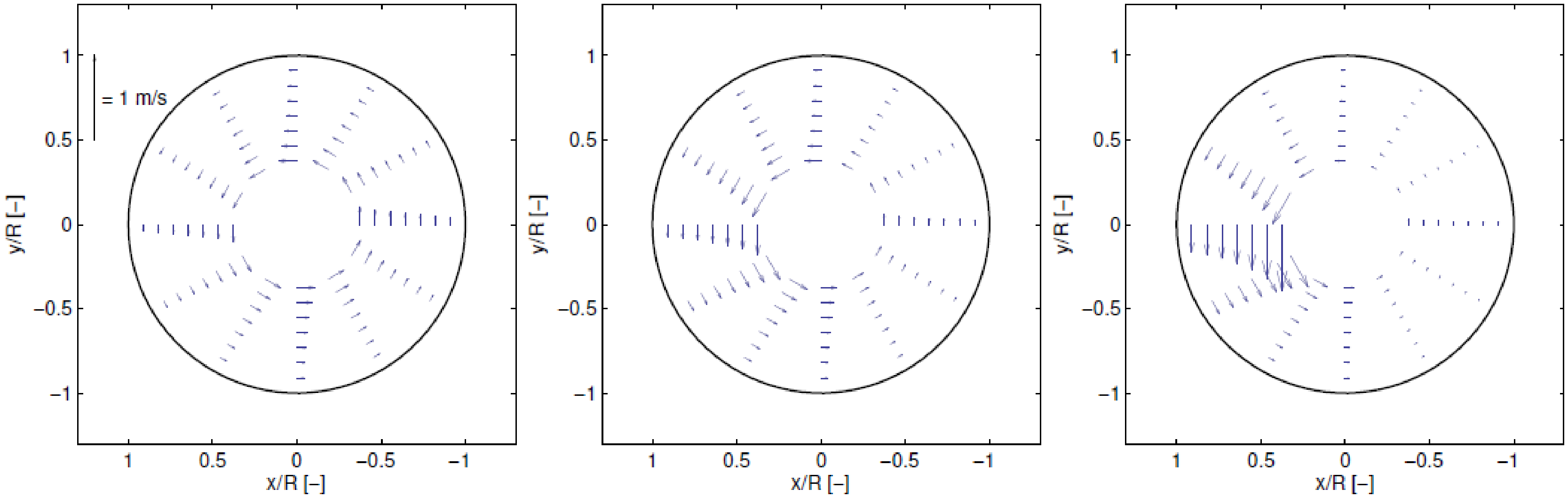
4. Longitudinal vorticity

Root vortex

$$u_{z,r} = \frac{\Gamma_r}{4\pi r(1-\cos\psi\sin\chi)}\sin\psi\sin\chi$$

(See also WEH)

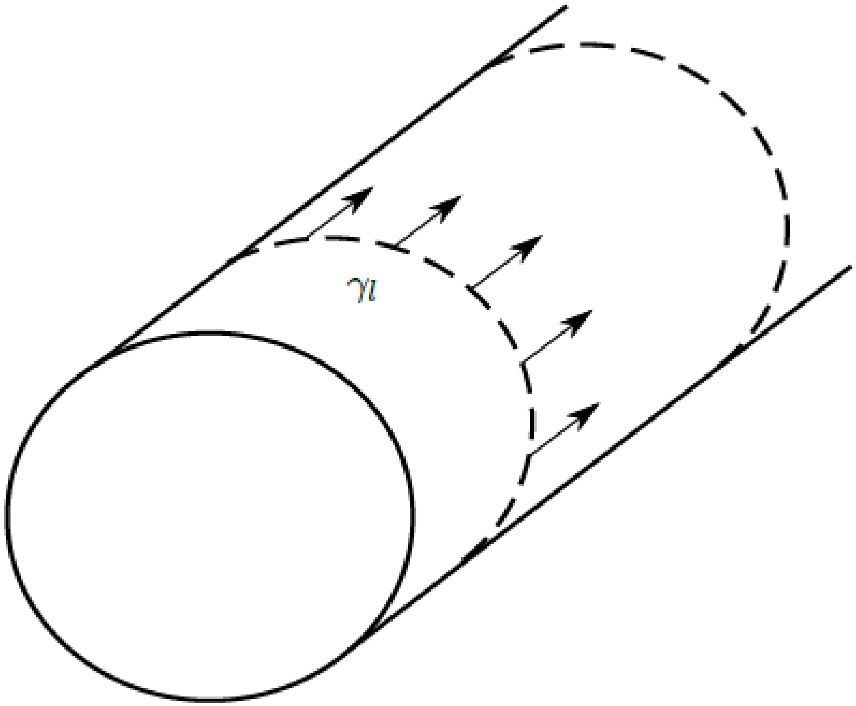
$$u_{\psi,r} = \frac{\Gamma_r}{4\pi r(1-\cos\psi\sin\chi)}\cos\chi = u_{\psi_r}(r=0,\chi=0)K_{\psi,r}(\psi,\chi)$$



(a) $\chi = 0°$ (b) $\chi = 30°$ (c) $\chi = 60°$

4. Longitudinal vorticity

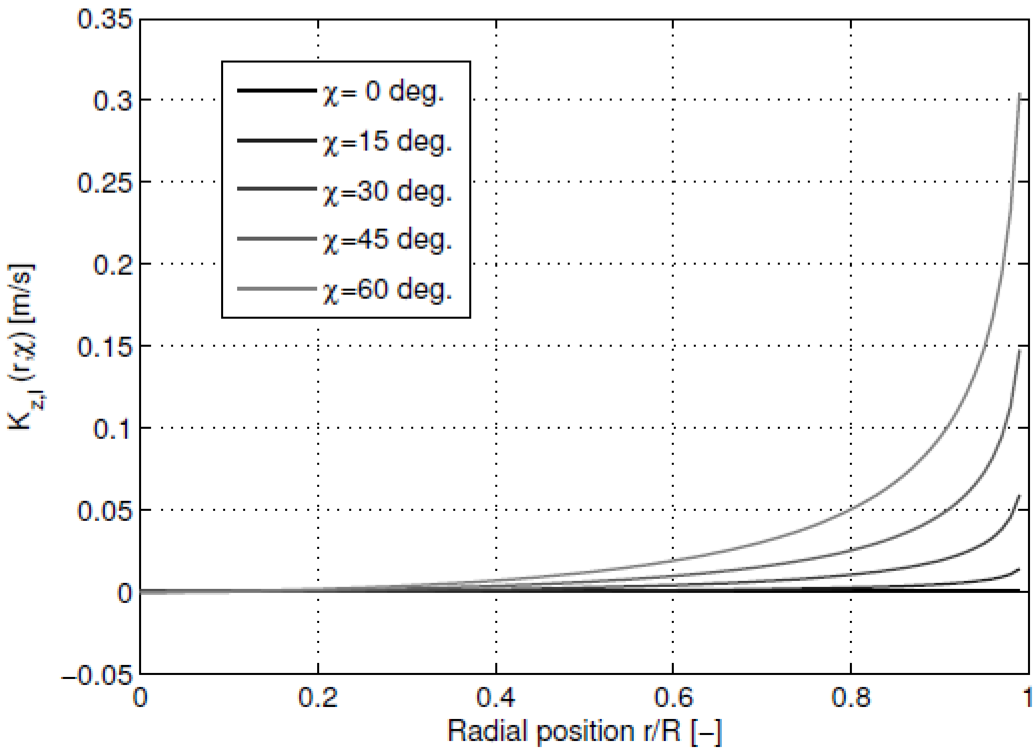
Tip-vortices



4. Longitudinal vorticity

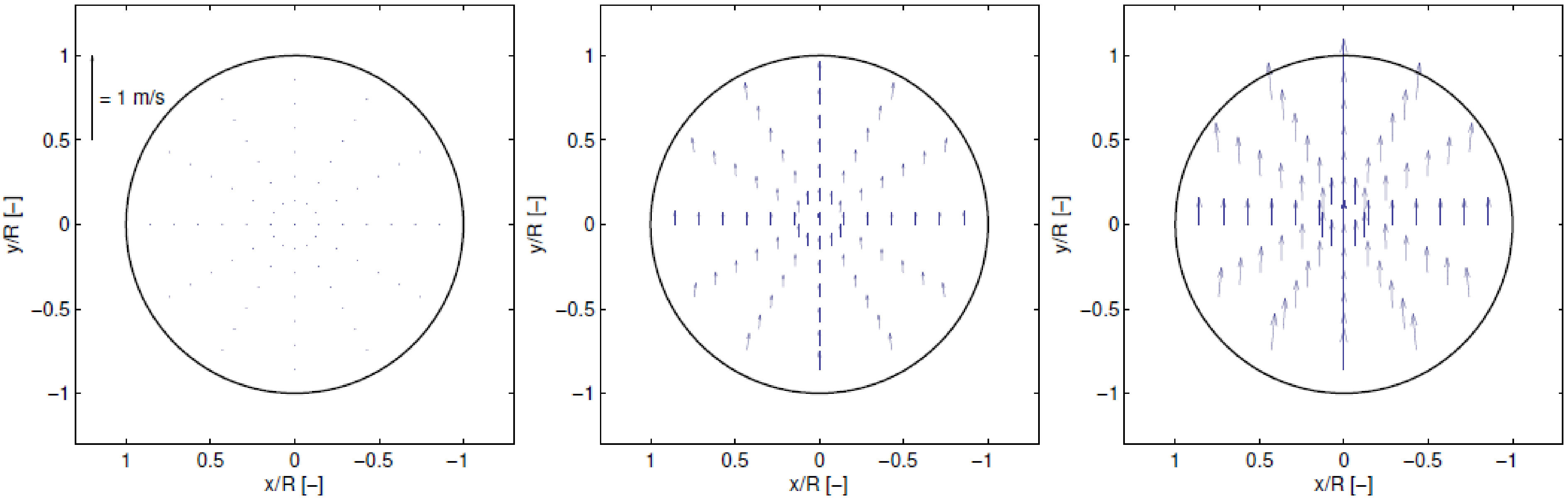
Tip-vortices – Axial component

$$u_{z,l}(r,\psi,\chi) = -\gamma_l \, K_{z,l}(r,\chi) \, \sin(2\psi)$$



4. Longitudinal vorticity

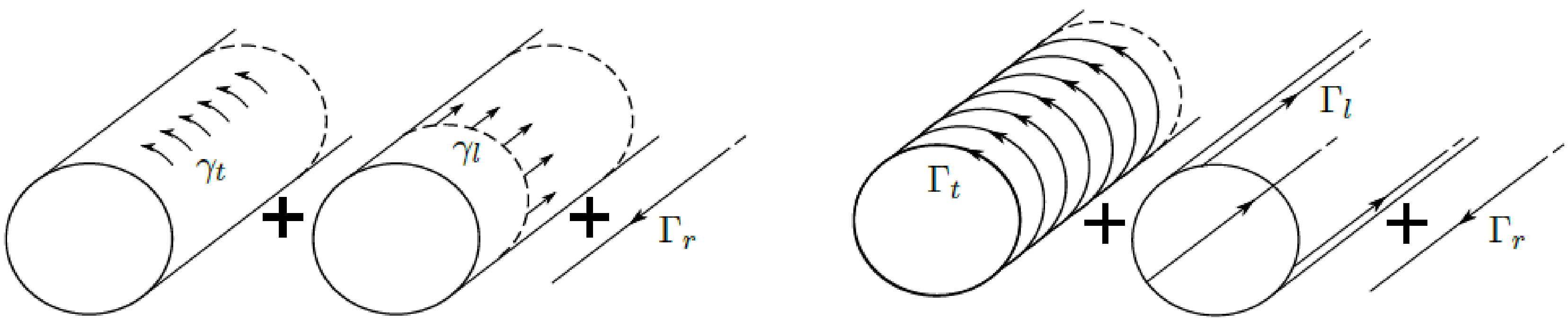
Tip-vortices – In-plane components



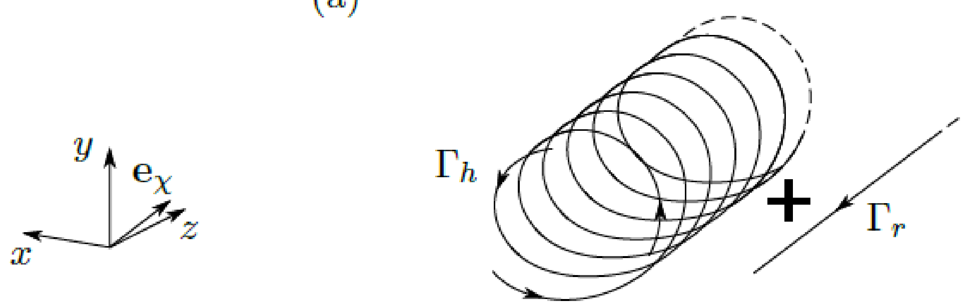
(a) $\chi = 0°$ (b) $\chi = 30°$ (c) $\chi = 60°$

5. Putting pieces together

Relating vorticity intensity



(a) (b)



(c)

$$\Gamma_b = \frac{\Gamma_{\text{tot}}}{n_B}$$

$$\mathbf{\Gamma_r} = -\Gamma_{\text{tot}}\mathbf{e}_\zeta$$

$$\Gamma_h = \Gamma_b$$

$$\mathbf{\Gamma_l} = \Gamma_b\mathbf{e}_\zeta$$

$$\mathbf{\Gamma_t} = -\Gamma_b\mathbf{e}_\psi$$

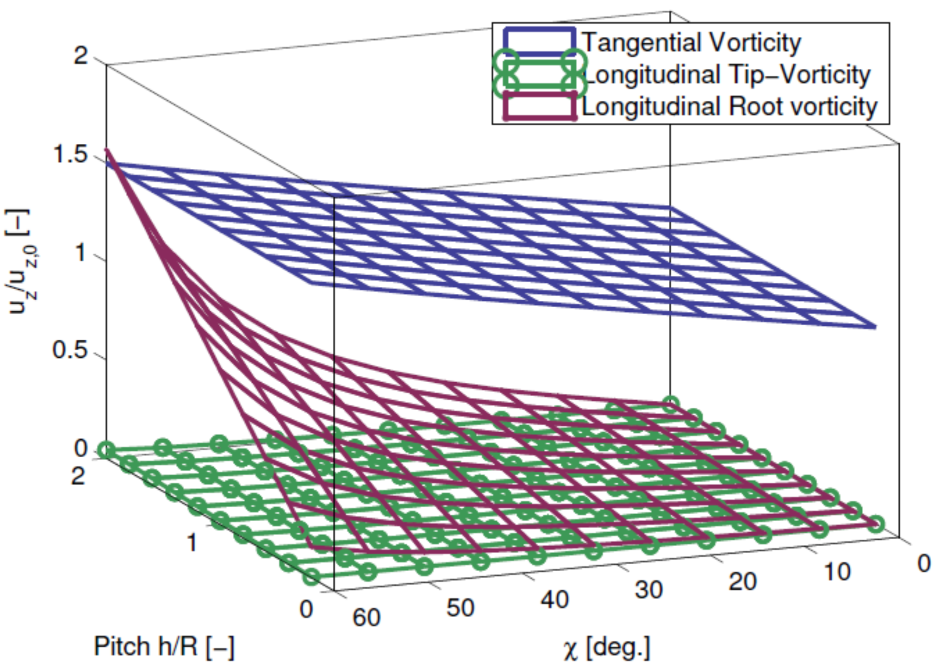
$$\gamma_{\mathbf{t}} = -\frac{\Gamma_{\text{tot}}}{h/\cos\chi}\mathbf{e}_\psi$$

$$\gamma_{\mathbf{l}} = \frac{\Gamma_{\text{tot}}}{2\pi R}\mathbf{e}_\zeta$$

5. Putting pieces together

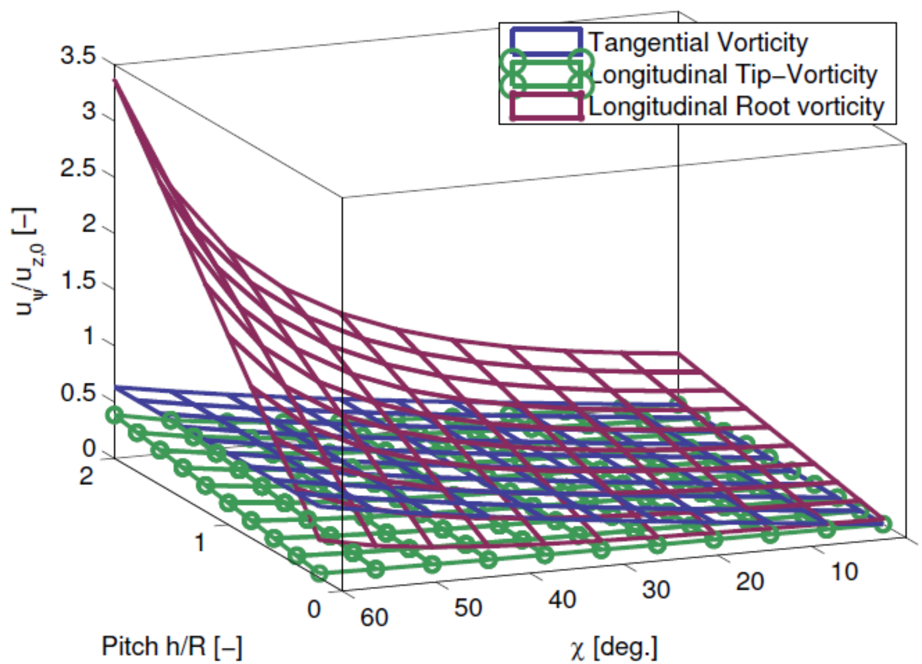
Amplitude comparison – Small pitch

Axial velocity



$u\_z,l\ /u\_z,0 = 0.5\%$

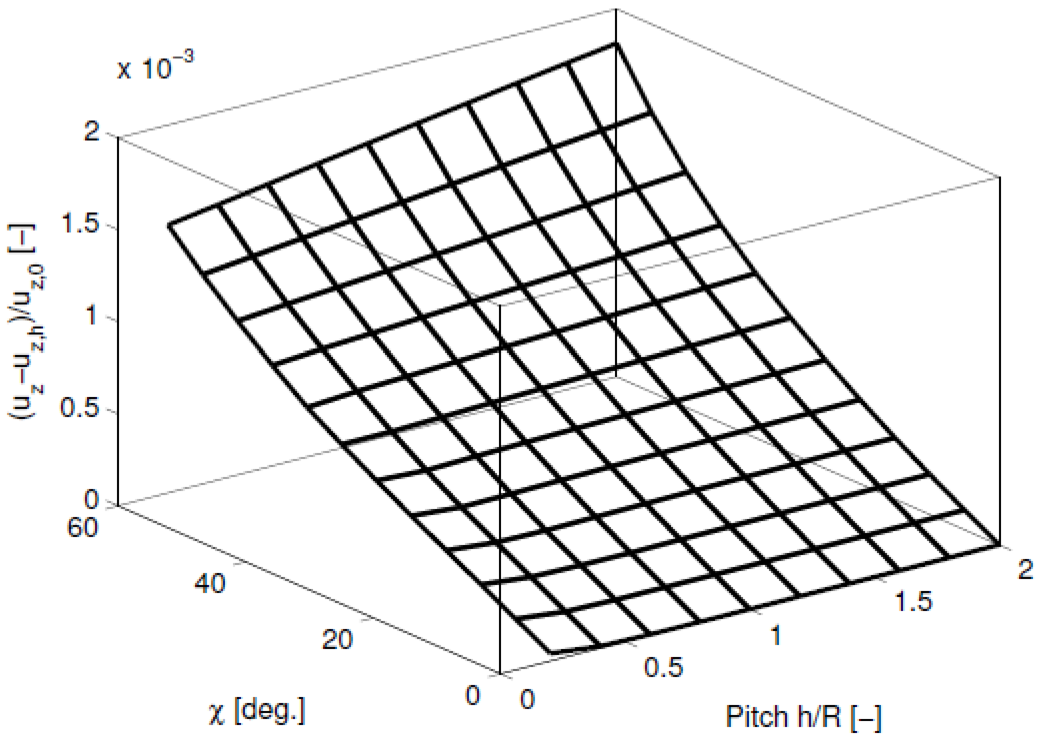
Tangential velocity



u_psi,l /u_z,0 = 4%

5. Putting pieces together

How good is this projection ?



Conclusions

- Full velocity field from longitudinal and tangential vorticity obtained with combined analytical and numerical integration
- Simple approximations or empirical formulae can be derived for implementation in BEM codes
- Influence of longitudinal tip-vorticity is small compared to other components

Future work

- Implementation in BEM
- Comparison with free-wake vortex code and experiments
- Relaxing infinite number of blade assumption (tip-losses)
- Relaxing the constant circulation hypothesis

Thank you for your attention